2022

B.A./B.Sc.

Fourth Semester

CORE - 8

PHYSICS

Course Code: PHC 4.11

(Mathematical Physics – III)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then prove that

$$x^p y^q + \frac{1}{x^p y^q} = 2\cos(p\theta + q\phi)$$ 5

(b) Find the expression $\frac{(\cos\theta + i\sin\theta)^8}{(\sin\theta + i\cos\theta)^4}$ in the form of $x + iy$. 4

(c) If ω is a cube root of unity, prove that $(1-\omega)^6 = -27$. 5

2. (a) Show that the real function $e^x(\cos y + i\sin y)$ is an analytic function. Find its derivative. 4

(b) Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$ along the real axis from $z = 0$ to $z = 2$ and then along a line parallel to y-axis from $z = 2$ to $z = 2 + i$ 5

(c) Evaluate the integral $\int_C \log z\, dz$, where C is the unit circle $|z| = 1$. 5

UNIT–II

3. (a) Evaluate: $\int_C \frac{z}{\left(z^2-3z+2\right)}dz$ by using Cauchy's integral formula, where C is the circle $|z-2|=\frac{1}{2}$. 4

(b) Find the first three terms of the Taylor series expansion of $f(x)=\frac{1}{z}$ about $z=2$. Find the region of convergence. 5

(c) Evaluate $\oint_C \frac{2z^2+5}{(z+2)^3\left(z^2+4\right)}dz$, where C is the square with the vertices at $1+i, 2+i, 2+2i, 1+2i$. 5

4. (a) Determine the poles and residue at simple pole of the function

$$f(z)=\frac{z^2}{(z-1)^2(z+2)}$$ 4

(b) Using Residue theorem, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$, where C is the circle $|z|=\frac{3}{2}$ 5

(c) Evaluate $\int_0^{2\pi}\frac{d\theta}{5-4\sin\theta}$ 5

UNIT–III

5. (a) Find the Fourier cosine integral of the function e^{-ax}. Hence show that $\int_0^\infty \frac{\cos\lambda x}{\left(\lambda^2+1\right)}d\lambda=\frac{\pi}{2}e^{-x}, x\geq 0$ 4

(b) Find the Fourier transform of $f(x)=\begin{cases}1-x^2 \ if \ |x|\le 1\\ 0 \ if \ |x|>1\end{cases}$ and use it to

evaluate $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$ 7

(c) Find the Fourier sine transform of e^{-ax}. 3

6. (a) State and prove the Convolution theorem on Fourier transform. 4

(b) If $F_1(s)$ and $F_2(s)$ are Fourier transforms of $f_1(x)$ and $f_2(x)$ respectively, show that $F\left[af_1(x)+bf_2(x)\right]=aF_1(s)+bF_2(s)$. 3

(c) Let $F(s)$ is the complex Fourier transform of $f(x)$. Show that

$F\{f(x)\cos ax\}=\frac{1}{2}\left[F(s+a)+F(s-a)\right]$. 4

(d) Show that $F\{f'(x)\}=is\,F(s)$, if $f(x)\to 0$ as $x\to\pm\infty$ 3

UNIT–IV

7. (a) Find the Laplace transform of $f(t)=\begin{cases}t^2, & 0<t<2\\ t-1, & 2<t<3\\ 7, & t>3\end{cases}$ 5

(b) Solve the initial value problem.

$y''+2y'-3y=3,\ y(0)=4;\ y'(0)=-7$ 6

(c) Find $L^{-1}\left[\frac{1}{s^2\left(s^2+4\right)}\right]$ using Laplace transform of integral. 3

8. (a) Find the Laplace transform of the triangular wave function of period $2C$ given by $f(t)=\begin{cases}t, & 0<t\le C\\ 2C-t, & C<t<2C\end{cases}$. 5

(b) Find the solution of the initial value problem.

$y''+ty'-2y=6-t, y(0)=0; y'(0)=1$ 6

(c) Find the Inverse Laplace transform of $\frac{2(s+1)}{(s^2+2s+2)^2}$. 3

UNIT–V

9. (a) Solve $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ for $x \geq 0, t \geq 0$ under the given conditions $u=u_0$ at $x=0, t>0$ with initial conditions $u(x,0)=0, x \geq 0$ 7

(b) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, $x>0, t>0$ subjected to the conditions

(i) $u=0$ when $x=0, t>0$

(ii) $u=\begin{cases}1, & 0<x<1 \\ 0, & x \geq 1\end{cases}$, when $t=0$

(iii) $u(x,t)$ is bounded 7

10. (a) A resistance *R* in series with inductance (*L*) is connected with emf *E*(*t*). The current *i* is given by $L\frac{di}{dt}+Ri=E(t)$. If the switch is connected at $t=0$ and disconnected at $t=a$, find the current *i* in terms of *t*. 7

(b) A particle moves in a line so that its displacement *x* from a fixed point O at any time *t* is given by $\frac{d^2x}{dt^2}+4\frac{dx}{dt}+5x=80\sin 5t$. Using Laplace transform, find its displacement at any time '*t*' if initially particle is at rest at $x=0$ 7