2022

B.A./B.Sc.

Fourth Semester

CORE - 8

MATHEMATICS

Course Code: MAC 4.11

(Numerical Methods)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

Note: Scientific, non-programmable calculators are allowed

UNIT–I

1. (a) Define an algorithm. Write an algorithm to determine if a positive integer is prime or not. 7

   (b) Find the sum of the numbers 0.1532, 15.45, 0.000354, 305.1,8.12,143.3, 0.0212, 0.643 and 0.1734, where each number is correct to the digits given. Estimate the absolute error in the sum. 5

   (c) If an approximate value of π is given by $x_1 = \frac{22}{7} = 3.142857$ and its true value is $x = 3.1415926$, find the absolute and relative errors. 2

2. (a) Define the term absolute error. Given $c = 15300 \pm 100$, find the maximum value of the absolute error in c^3. 4

   (b) Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its relative error. 3

   (c) Write an algorithm to compute the real roots of the quadratic equation $ax^2 + bx + c = 0$ 7

UNIT–II

3. (a) Apply secant method to determine a root of the equation $f(x) = \cos x - xe^x = 0$ by taking the initial approximations $x_0 = 0, x_1 = 1$. Perform three iterations. 7

   (b) Define Rate of convergence. Show that the Newton-Raphson's Method has second order convergence. 7

4. (a) Find the iterative methods based on Newton-Raphson method for finding $N^{1/3}$, where N is a positive real number. Apply the method to $N = 18$ to obtain the result, performing four iterations. 7

   (b) Find the interval in which the smallest positive root of the equation $x^3 - x - 4 = 0$ lie. Determine the root using the bisection method. Perform four iterations. 7

UNIT–III

5. (a) Solve the following system of equations using the Gauss elimination method with partial pivoting:

$$\begin{bmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 5

   (b) Prove that the Gauss Jacobi iteration method for the solution Ax = b converges to the exact solution for any initial vector if $\|H\| < 1$ 5

   (c) Show that the Gauss-Seidel method for solving the system of equations

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix}$$ diverges. 4

6. (a) Solve the system of equations Ax = b, where

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}, b = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$, using the LU decomposition

method. Take all the diagonal elements of L as 1. Also find A^{-1} 7

(b) Solve the system of equations:

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1$$

using the Gauss-Seidel method taking the initial approximation as $x^{(0)} = 0$ and perform three iterations. 7

UNIT–IV

7. (a) Derive the Lagrange linear interpolation formula. 4

(b) Taking the interval of difference as unity, evaluate the following (*a* and *b* are constants) 5

(i) $\Delta \tan(ax)$

(ii) $\Delta^n \left(e^{ax+b}\right)$

(c) Given that $y_{35.0} = 1175, y_{35.5} = 1280, y_{39.5} = 2180, y_{40.5} = 2420$, obtain a value for y_{40}. 5

8. (a) Establish the following relations: 5

(i) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$

(ii) $\Delta + \nabla \equiv \Delta / \nabla - \nabla / \Delta$

(b) For the following data, calculate the differences and obtain the Gregory-Newton forward difference polynomial. Interpolate at $x = 0.25$. 5

x	:	0.1	0.2	0.3	0.4	0.5
f(x)	:	1.40	1.56	1.76	2.00	2.28

(c) Find the missing term in the following table. 4

x	:	0	1	2	3	4
y	:	1	3	9	...	81

UNIT–V

9. (a) Find from the following table, the area bounded by the curve and the x-axis from $x = 7.47$ to $x = 7.52$ 5

x	: 7.47	7.48	7.48	7.50	7.51	7.52
f(x)	: 1.93	1.95	1.98	2.01	2.03	2.06

(b) Use Simpson's three-eight rule to obtain the value of

$$\int_0^{0.3} \left(1-8x^3\right)^{1/2} dx$$ 5

(c) Using Euler's method, solve the differential equation $y' = -y$ with the condition $y(0) = 1$. 4

10. (a) A river is 80 feet wide. The depth *d* (in feet) of the river at a distance *x* from one bank is given by the following table:

x :	0	10	20	30	40	50	60	70	80
d :	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross section of the river using these data. 7

(b) Given $\frac{dy}{dx} = 1 + y^2$, where $y = 0$ when $x = 0$, find $y(0.2), y(0.4)$ and $y(0.6)$ using the classical Runge-Kutta fourth order method. 7