2022

B.A./B.Sc.

Second Semester

GENERIC ELECTIVE – 2

STATISTICS

Course Code: STG 2.11

(Introductory Probability)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define correlation. Show that correlation is independent of change of origin and scale. 5

   (b) Obtain the normal equations for estimating *a* and *b* from the line of regression $Y = a + bX$. 5

   (c) What is regression coefficient? Show that correlation coefficient is the geometric mean between the regression coefficients. 2+2=4

2. (a) What is scatter diagram? Prove that: $\rho = 1 - \dfrac{6\sum_{i=1}^{n} di^2}{n(n^2-1)}$ 2+5=7

   (b) Show that the angle θ between the two lines of regression is given by

   $$\theta = \tan^{-1}\left\{\frac{1-r^2}{|r|}\left(\frac{\sigma_X\sigma_Y}{\sigma_X^2+\sigma_Y^2}\right)\right\}$$ 4

   (c) If $Z = aX + bY$ and r is the correlation coefficient between *X* and *Y*, show that $\sigma_z^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2abr\sigma_X\sigma_Y$ 3

UNIT–II

3. (a) Obtain the equation of the plane of regression of X_1 on X_2 and X_3 6

(b) If $1-R^2_{1.23}=(1-r^2_{12})(1-r^2_{13.2})$, deduce that:

(i) $R^2_{1.23}=r^2_{12}+r^2_{13}$, if $r_{23}=0$

(ii) $1-R^2_{1.23}=\dfrac{(1-\rho)(1+2\rho)}{(1+\rho)}$, provided all coefficient of zero order are equal to ρ.

(iii) if $R_{1.23}=0$, X_1 is uncorrelated with any of other variables, i.e., $r_{12}=r_{13}=0$ 3+3+2=8

4. (a) Derive the multiple correlation coefficients in terms of the total correlation coefficient between the pairs of variables. 6

(b) If r_{12} and r_{13} are given, show that r_{23} must lie in the range

$$r_{12}r_{13}\pm\sqrt{\left(1-r^2_{12}-r^2_{13}+r^2_{12}r^2_{13}\right)}$$ 4

(c) Write the properties of residuals. 4

UNIT–III

5. (a) Define the following: 2×2=4
   (i) Probability mass function
   (ii) Probability density function

(b) A random variable X is distributed at random between the values 0 and 1 so that its probability density function is, $f(x)=kx^2(1-x^3)$, where k is a constant. Find the value of k. Using this value of k, find its mean and variance. 4

(c) State and prove Chebyshev's inequality. 6

6. (a) Define the following: 2×2=4
   (i) Continuous distribution function
   (ii) Weak law of large numbers

(b) (i) Find the expectation of the number on a die when thrown. 2

(ii) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them. 3

(c) Given the following table: 1+1+1+2=5

x	:	–3	–2	–1	0	1	2	3
$p(x)$	:	0.05	0.1	0.3	0	0.3	0.15	0.1

Compute:

(i) $E(X)$ (ii) $E(2X \pm 3)$

(iii) $E(4X+5)$ (iv) $V(X)$

UNIT–IV

7. (a) Define binomial distribution. Obtain mean, variance and moment generating function of binomial distribution. 1+1+2+2=6

(b) Ten coins are thrown simultaneously. Find the probability of getting at least seven heads. 4

(c) Define geometric distribution. Hence compute its mean and variance. 1+1+2=4

8. (a) Define Poisson distribution. Obtain its mean, variance and moment generating function. 1+1+2+2=6

(b) A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality? 4

(c) Derive Poisson distribution as a limiting form of a binomial distribution. 4

UNIT–V

9. (a) Define normal distribution. Discuss chief characteristics of normal distribution. 1+6=7

(b) Prove that a linear combination of n independent normal variates is a normal variate. Obtain the moment generating function of normal distribution. 5+2=7

10. (a) Define uniform distribution. Obtain its mean, variance, mgf and characteristic function. 1+1+1+2+2=7

(b) Define exponential distribution. Hence obtain its mean and variance. 1+1+1=3

(c) Show that an exponential distribution "lacks memory", i.e., if X has an exponential distribution then for every constant $a \geq 0$, one has

$P\left(Y \leq \frac{x}{X} \geq a\right) = P(X \leq x)$ for all x, where $Y = X - a$. 4