2022

B.A./B.Sc.

Second Semester

GENERIC ELECTIVE – 2

MATHEMATICS

Course Code: MAG 2.11

(Algebra)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) If $x, y \in \mathbb{R}$ then prove that $|x-y| \geq ||x|-|y||$ — 3

(b) Show that $\frac{n+1}{2n+1} \to \frac{1}{2}$ as $n \to \infty$ — 3

(c) Prove that a sequence $\{x_n\}$ can have at most one limit. — 6

(d) Find the n^{th} term of $\left\{\frac{1}{2}, \frac{2}{5}, \frac{4}{10}, \cdots\right\}$ — 2

2. (a) Show that the sequence $\{x_n\}$ where $x_n = \left(1+\frac{1}{n}\right)^n$ is monotone increasing and bounded and hence converges. — 6

(b) Solve $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n}{n^2}\right)$ — 3

(c) Show that $\left\{\sqrt[3]{n-2}\right\}$ diverges to infinity. — 3

(d) Find the n^{th} term of $\left\{\frac{2}{3\cdot 4}, \frac{3}{4\cdot 5}, \frac{4}{5\cdot 6}, \cdots\right\}$ — 2

UNIT–II

3. (a) Check the convergence or divergence of the series

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\cdots+\frac{1}{n(n+1)(n+2)}+\cdots$$ 6

(b) Prove by comparison test that the series

$\frac{1}{1\cdot 3}+\frac{2}{3\cdot 5}+\frac{3}{5\cdot 7}+\frac{4}{7\cdot 9}+\cdots$ is divergent. 3

(c) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$ 5

4. (a) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if $p>1$ and divergent for $0<p\le 1$. 6

(b) Show that the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ is absolutely convergent. 3

(c) Use integral test to show that the series $\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots$ is convergent or divergent. 5

UNIT–III

5. (a) Prove that, in a polynomial equation with real coefficient, imaginary roots occur in pairs. 4

(b) Solve the equation $x^4-7x^3+27x^2-47x+26=0$, given that one of its root is $2-3i$. 5

(c) Solve the following reciprocal equation.

$x^5-5x^4+9x^3-9x^2+5x-1=0$ 5

6. (a) Solve the following equation by Cardon's method.

$x^3-15x^2-33x+847=0$ 6

(b) Transform the equation into an equation lacking the third term.

$x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$ 6

(c) If α, β, γ be the roots of $x^3 + x + 1 = 0$, prove that

$\alpha^2 + \beta^2 + \gamma^2 = -2$ 2

UNIT–IV

7. (a) Define an operation * on $\mathbb{R}$ in the following way.

$\forall x, y \in \mathbb{R}, x * y = xy + 1$.

Show that the operation is commutative but not associative. 4

(b) Show that the set of integers form an abelian group with respect to the binary operation * defined by $a * b = a + b + 1 \ \forall a, b \in \mathbb{Z}$ 4

(c) Let *G* be the additive group of all integers. Prove that the set of all multiples of integer by a fixed integer *m* is a subgroup of *G*. 4

(d) Consider the group $(\mathbb{Z}_4, +_4)$, where $\mathbb{Z}_4 = \{0, 1, 2, 3\}$. Find the order of each element. 2

8. (a) Show that the group $G = (\{1, 2, 3, 4, 5, 6\}, \times_7)$ is cyclic. How many generators of *G* are there? 4

(b) Let *G* be a group of nonzero complex numbers $(a + ib)$ such that $a, b \in \mathbb{R}$ under the operation of multiplication of complex numbers. Let $H = \{a + ib \in G / a^2 + b^2 = 1\}$. Show that *H* is a subgroup of *G*. 6

(c) If *H*, *K* are two subgroups of a group *G* then prove that *HK* is a subgroup of *G* if and only if *HK* = *KH*. 4

UNIT–V

9. (a) Prove that, if *G* is an abelian group, then for all $a, b \in G$ and all integers *n*, $(ab)^n = a^n b^n$ 4

(b) Let $H = \{I, (1\,2)\}$ and $K = \{I, (1\,3)\}$ be two subgroups of S_3. Find HK and using Lagrange's theorem prove that HK is not a subgroup of S_3 5

(c) Prove that any two left (right) cosets of a subgroup are either identical or disjoint. 5

10. (a) Find all the cosets of $3\mathbb{Z}$ in the group $(\mathbb{Z}, +)$ 4

(b) Prove that every group of prime order is cyclic. 4

(c) Express $\psi = (1\,2\,3\,4\,5)(1\,2\,3)(4\,5) \in S_5$ as the product of disjoint cycles and find the order. 4

(d) Find the number of generators of the group $(\mathbb{Z}_{12}, +_{12})$ 2