2022

B.A./B.Sc.

Second Semester

CORE – 4

STATISTICS

Course Code: STC 2.21

(Algebra)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Find the roots of the equation $x^3 - 3x^2 - 16x + 48 = 0$, when sum of two roots is zero. 4

   (b) Calculate the values of the following symmetric function for the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, whose roots are $\alpha, \beta, \gamma, \delta$. 2×3=6

   (i) $\sum \alpha^2$ (ii) $\sum \alpha^2\beta$ (iii) $\sum \frac{1}{\alpha^2}$

   (c) Find the quotient and remainder when $4x^3 + 3x^2 + 2x - 4 \div (2x+1)$, applying synthetic division. 4

2. (a) Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$, given that two of whose roots are in the ratio $3:2$. 5

   (b) Prove that in an equation with rational coefficients, the roots which are quadratic surds occur in conjugate pairs. 5

   (c) Divide $6x^3 - 11x^2 + 13x - 7$ by $(3x-4)$, using synthetic division. 4

UNIT–II

3. (a) Define nilpotent matrix. Show that the $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 1 & 3 \\ 1 & -3 & 0 \end{bmatrix}$ is nilpotent and find its index. 1+3=4

(b) Define transpose of a matrix. If A^T and B^T are the transposes of A and B respectively, then show that $(AB)^T = B^T A^T$. 2+4=6

(c) Find the inverse of the matrix A and solve the equations $AX = B$,

where, $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ 4

4. (a) Define trace of a matrix. If A and B are two square matrices of order n, then show that $\mathrm{tr}(AB) = \mathrm{tr}(BA)$. 2+4=6

(b) Show that every square matrix is uniquely expressible as the sum of a symmetric and a skew-symmetric matrix. 4

(c) Show that the necessary and sufficient condition for a square matrix A to possess the inverse is $|A| \neq 0$. 4

UNIT–III

5. (a) Define minors and cofactors of a determinant. Show that if any two rows (or two columns) of a determinant are interchanged, the value of the determinant is multiplied by (–1). 4+3=7

(b) Show that $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$, where ω is one of the imaginary cube roots of unity. 3

(c) Solve by Cramer's rule:

$$x + 2y + 3z = 6$$
$$2x + 4y + z = 7$$
$$2x + 2y + 9z = 14$$
4

6. (a) Show that if in a determinant each element of any row (or column) consists of the sum of two terms, then the determinant can be expressed as the sum of two determinants of same order. 3

(b) Prove that:

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$ 4

(c) Show that the value of the determinant of a skew symmetric matrix of odd order is always zero. 3

(d) Solve by Cramer's rule:

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$ 4

UNIT–IV

7. (a) Define echelon form of a matrix. Show that the rank of the transpose of a matrix is the same as that of the original matrix. 2+4=6

(b) Find the rank of the matrix $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing it to normal form. 8

8. (a) Define elementary matrix. Show that interchange of a pair of rows does not change the rank of a matrix. 2+4=6

(b) Find the inverse of the matrix $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ by using elementary transformation. 8

UNIT–V

9. (a) Define subspace. Prove that a subset W of a vector space $V(F)$ is a subspace of V, iff $\forall \alpha, \beta \in W$ and $a, b \in F \Rightarrow a\alpha + b\beta \in W$. 1+4=5

(b) Let U be the set of all vectors of the form $\begin{bmatrix} 2r - s \\ 2 \\ r + s \end{bmatrix}$ where $r, s \in \mathbb{R}$.

Is U a subspace of $\mathbb{R}^3$? Justify. 4

(c) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$.

Hence, find A^{-1}. 5

10. (a) Find whether the following vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix}$ are linearly dependent. 5

(b) State and prove Cayley-Hamilton theorem. 1+5=6

(c) Determine the characteristic roots of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ 4