**2022**

**B.A./B.Sc.**

**Second Semester**

CORE –- 4

# MATHEMATICS

*Course Code: MAC 2.21*

(Differential Equations)

*Total Mark: 70* *Pass Mark: 28*

*Time: 3 hours*

*Answer* five *questions, taking* one *from each unit.*

## UNIT–I

1. (a) Determine the constant $A$ such that the equation $\left(Ax^2y+2y^2\right)dx+\left(x^3+4xy\right)dy=0$ is exact and solve the resulting exact equation. 2+2=4

 (b) Solve: $\left(x^2y-2xy^2\right)dx=\left(x^3-3x^2y\right)dy$ 5

 (c) Solve: $x\dfrac{dy}{dx}+y=y^2\ln x$ 5

2. (a) Solve: $\sqrt{1+x^2}\sqrt{1+y^2}\,dx+xydy=0$ 4

 (b) Solve: $\dfrac{dy}{dx}=\dfrac{x-y+3}{2(x-y)+5}$ 5

 (c) Solve: $\left(1+x^2\right)\dfrac{dy}{dx}+2xy-4x^2=0$ 5

## UNIT–II

3. (a) Polonium-210 has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg.

 (i) Find a function that models that amount of sample remaining at time *t*. 3

(ii) Find the mass remaining after one year. 2

(iii) How long will it take for the sample to decay to a mass of 200 mg? 2

(b) In a fish farm, fishes are harvested at a constant rate of 2100 fish per week. The per capita death rate for the fish is 0.2 fish per day per fish and per capita birth rate is 0.7 fish per day per fish.

(i) Write down the word equation and differential equation for the rate of change of fish population. 5

(ii) Find equilibrium fish population. 2

4. (a) By using compartmental diagram, develop a mathematical model that describes drug assimilation into the blood in case of a single cold pill. 7

(b) A culture starts with 10,000 bacteria, and the number doubles every 40 minutes.

(i) Find a function that models the number of bacteria in time *t*. 4

(ii) After how much time will there be 50,000 bacteria? 3

**UNIT–III**

5. (a) Show that $x^2$ and $\frac{1}{x^2}$ are linearly independent solutions of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$. Also find the solution that satisfies the conditions $y(2)=3, y'(2)=-1$. 4+2=6

(b) If $y_1$ and $y_2$ are two solutions of $\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=0$ then prove that $y=c_1y_1+c_2y_2$ is also a solution of the given differential equation for any arbitrary constants $c_1$ and $c_2$. 4

(c) If $W(x)$ is the Wronskian of two functions $\phi_1(x)$ and $\phi_2(x)$, show that $\phi_1 W\left(\frac{\phi_2}{\phi_1},\phi_1\right)+\phi_2 W\left(\frac{\phi_1}{\phi_2},\phi_2\right)=\frac{d}{dx}(\phi_1\phi_2)$. 4

6. (a) Given that $y = x$ is a solution of $\left(x^2+1\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, find a linearly independent solution by reducing the order. 6

(b) If $W(x)$ is the Wronskian of two functions $\phi_1(x)$ and $\phi_2(x)$ where $\alpha$ is a constant, show that

$W(\phi_1+\alpha, \phi_2+\alpha) = W(\phi_1, \phi_2) + \alpha\frac{d}{dx}(\phi_1 - \phi_2)$. 4

(c) Show that $e^{2x}$ and $e^{3x}$ are linearly independent solutions of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ in the interval $-\infty < x < \infty$. Find the solution that satisfies the conditions $y(0) = 2, y'(0) = 3$. 2+2=4

**UNIT–IV**

7. (a) Solve: $\left(D^2+7D+10\right)y = 0$, given that $y(0) = -4, y'(0) = 2$ 5

$\left[\text{Symbol } D \text{ stands for } \frac{d}{dx}\right]$

(b) Solve: $\left(D^2-4D+4\right)y = x^3$ 5

(c) Solve: $\left(D^2+2D+2\right)y = e^{-x}\sin x$ 4

8. (a) Solve: $\left(x^2D^2-3xD+4\right)y = 2x^2$ 5

(b) Solve $\left(D^2-6D+9\right)y = e^{3x}$ by using the method of undermined coefficients. 4

(c) Solve $\left(D^2-2D+1\right)y = \frac{e^x}{x^2+1}$ by using the method of variation of parameters. 5

## UNIT–V

9. (a) Define an equilibrium point. Determine the nature of the equilibrium point for the linear system $\frac{dx}{dt} = 3x + 4y, \frac{dy}{dt} = 3x + 2y$. 1+4=5

   (b) Develop a mathematical model to describe the predator-prey system and discuss the behaviour of its equilibrium points. 5+4=9

10. (a) A simple model describing jungle warfare with one army exposed to random fire and the other to aimed fire is given by coupled differential equations

    $\frac{dR}{dt} = -c_1 RB, \ \frac{dB}{dt} = -c_2 R$, where $c_1, c_2$ are positive constants.

    (i) Use chain rule to find a relation between $B$ and $R$, given the initial numbers of blue and red soldiers are $b_0$ and $r_0$ respectively.

    (ii) Given that initially both blue and red armies have 1000 soldiers and the constants $c_1$ and $c_2$ are $10^{-4}$ and $10^{-1}$ respectively. Determine how many soldiers are left if the battle is fought so that all the soldiers of one army killed. 4+3=7

    (b) The epidemic model for infectious disease is given by $\frac{dS}{dt} = -\beta SI$ and $\frac{dI}{dt} = \beta SI - \gamma I$, where the symbols have their usual meaning.
    Find a relation between $I$ in terms of $S$, given that the initial number of susceptible is $s_0$ and the initial number of contagious infective is $i_0$. Also sketch the phase-plan trajectories.
    [ ln 2 = 0.693147, ln 3=1.098612, ln 5=1.609438 ] 5+2=7

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