2022

B.A./B.Sc.

Second Semester

CORE – 3

STATISTICS

Course Code: STC 2.11

(Probability Distributions & Correlation Analysis)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) What is expectation? State and prove the addition rule of mathematical expectation. 1+3=4

   (b) A coin is tossed four times, let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X. By simple counting, derive the probability distribution of X and hence calculate the expected value of X. 4

   (c) Two random variables X and Y have the following joint probability density function:

$$f(x,y)=\begin{cases}2-x-y; & 0\le x\le 1, 0\le y\le 1\\ 0; & \text{otherwise}\end{cases}$$

   Find (i) Marginal probability density function of X and Y.

   (ii) Conditional density functions

   (iii) $Var(X)$ and $Var(Y)$ 1+2+2=6

2. (a) Define moments and moment generating function of a random variable X. If $M(t)$ is the mgf of a random variable X about the origin, show that the moment μ'_r is given by $\mu'_r=\left[\frac{d^r\mu(t)}{dt^r}\right]_{t=0}$

   1+2+2=5

(b) Find the moment generating function of the random variable whose moments are: $\mu'_r = (r+1)!2^r$ 3

(c) Let X and Y be two random variables each taking three values –1, 0 and 1, and having the joint probability distribution

X→ Y↓	-1	0	1	Total
-1	0	0.1	0.1	0.2
0	0.2	0.2	0.2	0.6
1	0	0.1	0.1	0.2
Total	0.2	0.4	0.4	1

(i) Show that X and Y have different expectations.

(ii) Prove that X and Y are uncorrelated.

(iii) Find $Var(X)$ and $Var(Y)$.

(iv) Given that $Y = 0$, what is the conditional probability distribution of X? 1½×4 =6

UNIT–II

3. (a) Define binomial distribution. State the physical condition for binomial distribution. 2+4=6

(b) Derive the Poisson distribution as a limiting case of binomial distribution. 8

4. (a) Obtain the moment generating function of binomial distribution and hence find the value of mean and variance of binomial distribution. 3+4=7

(b) If X and Y are independent Poisson variates such that $P(X=1) = P(X=2)$ and $P(Y=2) = P(Y=3)$.

Find the variance of $(X-2Y)$. 4

(c) If X and Y are independently distributed Poisson variates with parameters m and n respectively, then prove that $X - Y$ is not a Poisson variate. 3

UNIT–III

5. (a) Define uniform distribution with usual notations. Find mean, variance and moment generating function of uniform distribution. 6

(b) Write four properties of normal distribution. Show that even order moments about mean of normal distribution are given by $\mu_{2n} = 1.3.5......(2n-1)\sigma^{2n}$. Also, find the mgf of normal distribution. 2+4+2=8

6. (a) Define normal distribution. Find the normal distribution as a limiting form of binomial distribution. Prove that for normal distribution, the quartile deviation, the mean deviation and the standard deviation are approximately 10:12:15 2+4+3=9

(b) Show that odd order moments of normal distribution are zero. Define the gamma distribution. 3+2=5

UNIT–IV

7. (a) X and Y are two random variables with variances σ_X^2 and σ_Y^2 respectively and r is the coefficient of correlation between them.

If $U = X + KY$ and $V = X + \left(\frac{\sigma_X}{\sigma_Y}\right)Y$, find the value of K so that U and V are uncorrelated. 7

(b) What is regression coefficient? Show that correlation coefficient is the geometric mean between the regression coefficients. 1+2=3

(c) Fit a second degree equation $Y = a + bx + cx^2$ by the method of least square. 4

8. (a) Prove that Spearman's rank correlation coefficient is

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$$ 5

(b) Let the line of regression of Y on X be $Y = a + bx$. Obtain the normal equation for estimating a and b. 5

(c) Prove that the angle θ between the two lines of regression is given

by $\theta = \tan^{-1}\left\{\frac{1-r^2}{|r|}\left(\frac{\sigma_X \sigma_Y}{\sigma_X^2 + \sigma_Y^2}\right)\right\}$ 4

UNIT–V

9. (a) Show that $1 - R_{1.23}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2)$, and using the above result prove that:

   (i) $R_{1.23}^2 = r_{12}^2 + r_{13}^2$, if $r_{23} = 0$

   (ii) $1 - R_{1.23}^2 = \frac{(1-\rho)(1+2\rho)}{(1+\rho)}$, provided all coefficient of zero order are equal to ρ. 3+3+3=9

   (b) Explain the properties of residuals. 5

10. (a) Show that the partial correlation coefficient between X_1 and X_2 is

$$r_{12.3} = \frac{\operatorname{cov}(X_{1.3}, X_{2.3})}{\sqrt{\operatorname{var}(X_{1.3})\operatorname{var}(X_{2.3})}}$$ 6

   (b) Write the properties of multiple correlation coefficients. 3

   (c) Derive the equation of the plane of regression of X_1 on X_2 and X_3 5