2021
M.Sc.
Third Semester
DSE – 01
MATHEMATICS
Course Code: MMAD 3.11
(Classical Mechanics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Derive Lagrange's equations of motion from D'Alembert's principle. 7

   (b) Obtain the equation of motion of a system of two masses, connected by an inextensible string passing over a small smooth pulley. 7

2. (a) Discuss the homogeneity of space and conservation of linear momentum. 6

   (b) A cylinder of radius a and mass m rolls down an inclined plane making an angle θ with the horizontal. Set up the Lagrangian and find the equation of motion. 8

UNIT–II

3. (a) Derive Hamilton's equations of motion in Cartesian coordinates. 7

   (b) Discuss the motion of a particle in a central force field. 7

4. (a) Describe the motion of a particle of mass m constrained to move on the surface of a cylinder of radius a and attracted towards the origin by a force which is proportional to the distance of the particle from the origin. 7

   (b) For a system with the Lagrangian $L = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_1\dot{q}_2 + \dot{q}_2^2\right) - V(q)$,

   show that the Hamiltonian is $H = \frac{2}{3}\left(p_1^2 - p_1 p_2 + p_2^2\right) + V(q)$. 7

UNIT–III

5. a) Derive Euler-Lagrange equation using the variational method. 7
   (b) State and solve the Brachistochrone problem. 7

6. (a) Deduce Hamilton's equations of motion form modified Hamilton's principle. 7
   (b) A curve AB, having end points A($x_1, y_{1)}$ and B(x_2, y_2), is revolved about x-axis so that the area of the surface of revolution is minimum.

   Show that $S = 2\pi \int_{x_1}^{x_2} y\sqrt{1+y'^2}\,dx$. Obtain the differential equation of the curve and prove that the curve represents a catenary. 7

UNIT–IV

7. (a) Show that the following transformations are canonical: 3+4=7

   (i) $Q = p\tan q, P = \log(\sin p)$

   (ii) $q = P^2 + Q^2, p = \frac{1}{2}\tan^{-1}\left(\frac{P}{Q}\right)$

   (b) Prove that the transformation $P = 2\left(1+\sqrt{q}\cos p\right)\sqrt{q}\sin p$ and $Q = \log\left(1+\sqrt{q}\cos p\right)$ is canonical. Find the generating function of this transformation. 5+2=7

8. (a) Discuss the invariance of Poisson bracket with respect to canonical transformations. 7
   (b) State and prove the Jacobi's identity. 7

UNIT–V

9. (a) Discuss Eulerian angles in detail. 10
   (b) Calculate the inertia tensor for the system of four-point masses 1gm, 2gm, 3gm, and 4gm, located at the points (1, 0, 0), (1, 1, 0), (1, 1, 1) and (1, 1, –1)cm. 4

10. (a) Discuss the force-free motion of a symmetrical top. 9

(b) If T be the kinetic energy, $\mathbf{G}$ be the external torque about the instantaneous axis of rotation and ω the angular velocity, then prove that

$$\frac{dT}{dt} = \mathbf{G}.\omega$$ 5
