2021
M.Sc.
Third Semester
CORE – 09
MATHEMATICS
Course Code: MMAC 3.11
(Numerical Analysis)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Write a short note on error in numerical analysis. 3
   (b) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $f(x) \equiv x^3 - 5x + 1 = 0$. 3
   (c) Compute a root of the equation $x^5 - 3x^2 - 100 = 0$ correct to three decimal places using Newton-Raphson method. 4
   (d) Discuss the rate of convergence of Regula-Falsi method. 4

2. (a) What is an iterative method? What are the criterion for termination of iterative methods? 2
   (b) Perform five iterations of the Muller's method to find the root of the equation $f(x) \equiv \cos x - xe^x = 0$ using the initial approximation $x_0 = -1, x_1 = 0$ and $x_2 = 1$. 6
   (c) Find the root of the equation $f(x) \equiv x^4 - x + 10 = 0$ correct to five decimal places using multipoint iteration method. (take $x_0 = 1.5$) 6

UNIT–II

3. (a) Solve the system of equations by Cramer's rule. 3

$$x + y + z = 1$$
$$2x + 3y - z = 6$$
$$3x + 5y + 3z = 4$$

(b) Solve the following system of equations 4

$$x + y + z = 6$$
$$2x + (3 + \varepsilon)y + 4z = 20$$
$$2x + y + 3z = 13$$

using Gauss elimination method, where ε is small such that $1 \pm \varepsilon \approx 1$.
7

(c) Using Cholesky or partition method to find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 1 & -1 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & -2 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$ 7

4. (a) Solve the system of equations 6

$$3x - 2y = 5$$
$$-x + 2y - z = 0$$
$$-2y + z = -1$$

using Gauss-Seidel method. Assume suitable approximation and perform four iterations.

(b) Find the largest and the smallest eigenvalue and its corresponding eigenvector of the matrix (only three iterations each)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

using power method. 8

UNIT–III

5. (a) Using Newton's divided difference formula, find $y(10)$ given that

$y(5)=12, y(6)=13, y(9)=14, y(11)=16$ 4

(b) The following data represents the function $f(x)=e^x$

x	1	1.5	2	2.5
$f(x)$	2.7183	4.4817	7.3891	12.1825

Estimate the value of $f(2.25)$ using Newton's backward difference interpolation. Compare with the exact value. Obtain the bound on the truncation error. 6

(c) Prove the following: 4

(i) If $f(x)=e^{ax}$, show that $\Delta^2 f(x)=\left(e^{ah}-1\right)^n e^{ax}$

(ii) $\Delta+\nabla=\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}$

(iii) $(1+\Delta)(1-\nabla)=1$

(iv) $\sqrt{1+\delta^2\mu^2}=1+\left(\frac{1}{2}\right)\delta^2$

6. (a) Construct the Hermite interpolation polynomial that fits the data 7

x	$f(x)$	$f'(x)$
2	29	50
3	105	105

Interpolate $f(x)$ at $x=2.5$

(b) Obtain the cubic spline interpolation for the data (taking $M_0=M_4=0$) 7

x	1	2	3	4	5
F(x)	1	0	1	0	1

UNIT–IV

7. (a) The velocity of a particle for 8 seconds at an interval of 2 seconds is given below. Find the initial acceleration using the entire data. 4

Time (sec)	0	2	4	6	8
Velocity (m/sec)	0	172	1304	4356	10288

(b) The following data represents the function $f(x) = e^{2x}$ 4

x	0	0.3	0.6	0.9	1.2
$f(x)$	1.0000	1.8221	3.3201	6.0496	11.0232

Find $f'(1.2), f''(0.9), f''(12)$ using Newton's backwards difference method. Compute the magnitudes of errors. 6

(c) Given the following data, find $y'(6)$ and $y''(6)$ 6

x	0	2	3	4	7	8
y	4	26	58	112	466	668

8. (a) Find the value of $\int_0^5 \frac{dx}{4x+5}$ using Simpson's1/3 rule and hence find the value of $\log_e 5$ (take $n = 10$). 7

(b) Evaluate $I = \int_0^2 \frac{dx}{3+4x}$ using the two point and three point Gauss quadrature. Compare with the exact solution. 7

UNIT–V

9. (a) Convert the following system of second order differential equation into a system of first order differential equation and write the corresponding initial value problem 4

$$y''' = e^x + y' + u' + u + y,\ y(1) = 3, y'(1) = 1$$

$$u'' = e^x + 9xu + 6u' + 9y' + 10y\ , u(1) = 1, u'(1) = 2$$

(b) Solve the following equation by Picards method and estimate y at $x = 0.25$ and 0.5

$$\frac{dy}{dx} = x + y, y(0) = 1$$ 4

(c) Find $y(0.1)$ using backward Euler method, if $y' = x^2 + y^2, y(0) = 1$ (use $h = 0.1$) 6

10. (a) Find the solution for the initial value problem 4

$$y' = x^2 - y^2, y(0) = 1, t \in [0, 0.6]$$

by Adams-Bashforth method of order three with $h = 0.1$. Determine the starting value using the third order Taylor series method.

(b) Solve the system of equations 5

$$u' = -3u + 2v, u(0) = 0$$

$$v' = 3u - 4v, v(0) = 0.5$$

with $h = 0.2$, use the classical Runge-Kutta fourth order method.

(c) For the initial value problem $y' = \dfrac{2y}{x}, y(1) = 2$, estimate $y(2)$ using Milne-Simpson method. Assume $h = 0.25$ 5