2021
M.Sc.
First Semester
CORE – 04
PHYSICS
Course Code: MPHC 1.41
(Statistical Mechanics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Explain the terms: phase space, ensemble, microstates and macrostates. 4
   (b) State and prove Boltzmann's theorem connecting entropy and probability. 5
   (c) What do you mean by partition function? Express entropy and Helmholtz free energy in terms of the partition function. 5
2. (a) Show that the mean energy for a canonical ensemble is given by 4

$$\overline{E} = KT^2 \frac{\partial}{\partial T} \ln Z$$

   (b) Define magnetic susceptibility. Obtain the condition for saturate state of thermodynamic systems. 5
   (c) Consider a system of N identical particles which can occupy energy level 0, E. Find the partition function and also find Helmholtz free energy, average energy, entropy and specific heat. 5

UNIT–II

3. (a) Write the partition function for a system of grand canonical ensemble. Use it to calculate the thermodynamics properties of an ideal gas. 7
   (b) Four identical particles can be in any of five states. What is the number of possible ways of distributing them in various states according to Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics? 3

(c) Explain the quantum Liouville's theorem. 4

4. (a) Explain density matrix. Obtain the equation for the postulates of random phase. 8

(b) Explain the conditions for Fermi-Dirac statistics and derive its energy distribution law. 6

UNIT–III

5. (a) Discuss the difference between MB, FD and BE statistics. Show that in the limiting case, both FD and BE statistics reduce to MB statistics. 7

(b) Explain Dulong-Petit's law of specific heat capacity. 4

(c) There are 2.54×10^{22} free electrons per cm^3 in sodium. Calculate its Fermi energy and Fermi velocity ($h = 6.63\times10^{-34}$ joule-sec, $m = 9.11\times10^{-31}$ kg, K=1.38×10^{-23} joule/k). 3

6. (a) Using necessary mathematical theory, prove that the specific heat of solid at low temperature varies as the cube of the absolute temperature. 7

(b) What is meant by black body radiation? Derive the Planck's radiation law and show that it reduces to Rayleigh-Jeans law for long wavelength. 7

UNIT–IV

7. (a) Derive the expression for the energy fluctuation in a canonical ensemble. 4

(b) Calculate the relative root mean square fluctuation in energy for a monoatomic ideal gas of energy $\frac{3}{2}NkT$ and $C_v = \frac{3}{2}Nk$. 3

(c) With the help of Einstein theory of Brownian motion, show that diffusion process is irreversible and keep on increasing with time. 7

8. (a) Discuss Fourier analysis of a random function. Hence derive Wiener-Khintchine theorem. 8

(b) Explain Langevin's theory of the Brownian motion and show that diffusion coefficient$=\frac{kT}{m\delta}$. 6

UNIT–V

9. (a) Derive Van der Waal's equation of state and find the thermodynamic coordinates of critical point V_C, P_C and T_C? 7

(b) Explain first and second order phase transitions with one example each. 4

(c) The coordinates of the triple point of water are $t = 0.0075°C$ and $p = 0.0060$ atmosphere. Calculate the slope of the ice line in atmos/°C. 3

10 (a) Derive the Ehrenfest's equations for the second order phase transition. 6

(b) Explain Ising Model. Use Bragg-William approximation method to obtain expressions for entropy and free energy under this model. 8