2021
M.Sc
First Semester
CORE – 03
PHYSICS
Course Code: MPHC 1.31
(Mathematical Physics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Find the Fourier transform of xe^{-ax^2} for $a > 0$. 3

   (b) Find the Fourier sine and cosine transform of x^{n-1}. 5

   (c) Solve the differential $2\dfrac{d^2y}{dt^2}+\dfrac{dy}{dt}-3y=e^{5it}$ using Fourier transform. 6

2. (a) Solve for $f(x)$ from the integral equation

$$\int_0^\infty f(x)\sin sx\,dx = \begin{cases} 1 \text{ for } 0 \le s<1 \\ 2 \text{ for } 1 \le s<2 \\ 0 \text{ for } s \ge 2 \end{cases}$$ 3

   (b) Find the Fourier sine transform of $f(x)=\dfrac{e^{-ax}}{x}$ and use it to evaluate

$$\int_0^\infty \tan^{-1}\left(\frac{x}{a}\right)\sin x\,dx \,.$$ 5

   (c) Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2}=\alpha^2\dfrac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t \ge 0$ with

   conditions $u(x,0)=f(x), \dfrac{\partial u}{\partial t}(x,0)=0$ and assuming $u, \dfrac{\partial u}{\partial t} \to 0$

as $x \to \pm\infty$ 6

UNIT–II

3. (a) Find the inverse Laplace transform of $\dfrac{s+1}{s^2-6s+25}$. 3

(b) State and prove convolution theorem. 5

(c) Solve the differential equation $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+5y=e^{-x}\sin x$, where $y(0)=0$, and $y'(0)=1$. 6

4. (a) State and prove first shifting theorem. 3

(b) Find the Laplace transform of $\sin\sqrt{t}$. Hence find $L\left(\dfrac{\cos\sqrt{t}}{\sqrt{t}}\right)$. 5

(c) An inductor of 3 henry is in series with a resistance of 30 ohms and an e.m.f of 150 volts. Assuming that the current is zero at t = 0, find the current time t > 0. 6

UNIT–III

5. (a) If Y^i is an independent function of the variables X^i and z^i are independent functions of Y^i and if $U^i = V^j \dfrac{\partial X^i}{\partial Y^j}$ and $V^i = W^j \dfrac{\partial Y^i}{\partial z^j}$ for $i, j = 1, 2, \ldots, n$, then show that $U^i = W^j \dfrac{\partial X^i}{\partial z^j}$. 3

(b) Show that tensor product of the tensors of the type (r,s) and $(\overline{r},\overline{s})$ is a tensor of the type $(r+\overline{r}, s+\overline{s})$. 5

(c) Show that the law of transformation of Christoffel's symbol possess group property. 6

6. (a) If a_{ij} is a second order rank covariant symmetric tensor and $\left|a_{ij}\right| = a$, then show that $\sqrt{a}$ is a scalar density. 3

(b) Show that $g_{ij}dx^i dx^j$ is an invariant. 3

(c) Derive the law of transformation of Christoffel symbol. 8

UNIT–IV

7. (a) Prove $\int_{-1}^{+1} P_m(x).P_n(x)\,dx = 0$. 6

(b) Prove the Rodrigue's formula for Legendre equation i.e.

$$P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n.$$ 8

8. (a) Show that Bessel's function $J_n(x)$ is an even function when n is even and is odd when n is odd. 6

(b) Derive the solution of Bessel's function of second kind of order n. 8

UNIT–V

9. (a) Suppose that $f : A \to B$ and $g : B \to C$. Then

(i) if f and g are both injective, then $g \circ f$ is also injective

(ii) if $g \circ f$ is injective, then f is injective. 4

(b) Show that the set of integers forms an abelian group under addition. 5

(c) Show that the group formed by the set $\left\{1, \omega, \omega^2\right\}$, ω being the cube root of unity i.e. $\omega^3 = 1$ is a cyclic group with respect to multiplication. 5

10. (a) Find the permutation group isomorphic to the group $\left(\{1,-1,i,-i\},\times\right)$ 4

(b) Explain the terms rotational, reflection and inversion symmetry with proper examples. How do we build up a point group? 6+4=10