2021
B.A./B.Sc.
Fifth Semester
Discipline Specific Elective – 2
MATHEMATICS
Course Code: MAD 5.21
(Boolean Algebra & Automata Theory)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

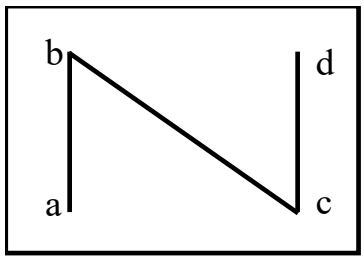
UNIT–I

1. (a) Prove that two finite ordered sets P and Q are order isomorphisms iff they can be drawn with identical diagram. 4

   (b) Draw the diagram of 2^4 and $M_3 \oplus M_4$ 3+2=5

   (c) Define order preserving and order embedding map with examples. Let $\varphi : P \to Q$ and $\psi : Q \to R$ be order preserving maps.Show that the composite map $\psi o \varphi$ given by $(\psi o \varphi)x = \psi(\varphi(x))$ for $x \in P$ is also order preserving map. 2+3=5

2. (a) Draw and label a diagram of the order sets $Q(P)$ of down sets for the ordered set P given by the diagram 4



   (b) Define lattices. Draw Hasse diagram of all lattices with six elements. 1+4=5

   (c) Let $(L, \wedge, \vee)$ be non-empty set equipped with two binary operations which satisfy the axioms of join and meet. Then 5

(i) Prove that $\forall a, b \in L$ we've $a \vee b = b$ iff $a \wedge b = a$

(ii) Define $\leq$ on L by $a \leq b$ if $a \vee b = b$ and prove that $\leq$ is ordered relation.

UNIT–II

3. (a) Prove that a lattice L is distributive iff cancelation rule holds. 4

(b) In a Boolean algebra B, show that $\forall x, y \in B$

$$x \leq y \Leftrightarrow x' \geq y' \Leftrightarrow x \wedge y' = 0 \Leftrightarrow$$
$$x' \vee y = 1 \Leftrightarrow x \wedge y = x \Leftrightarrow x \vee y = y$$

(c) Simplify the following Boolean polynomial to normal form. 5

(i) $xy + x'y + xy'$

(ii) $x\left(y+z\right)' + \left(xy + z'\right)x$

(iii) $\left(x + \left(x' + xy'\right)'\right)'$

(iv) $xy + yz + zx$

4. (a) Minimize the $xyz' + x'yz' + \left(x' + y'z'\right)'\left(x + y + z'\right)' + x\left(y+z\right)'$ using K-map and draw the contact diagram. 4

(b) Using Quine-McCluskey method minimize the Boolean polynomial $p = \sum\left(0,5,8,9,10,11,14,15\right)$. 5

(c) A motor is supplied by three generators where operation of each generator is monitored. Design a switching circuit to obtain the outputs satisfying the following conditions: 5

(i) A warning lamp lights up if one or two generator fails

(ii) An acoustic alarm is initiated if two or all three generators fails

UNIT–III

5. (a) Define a regular expression.

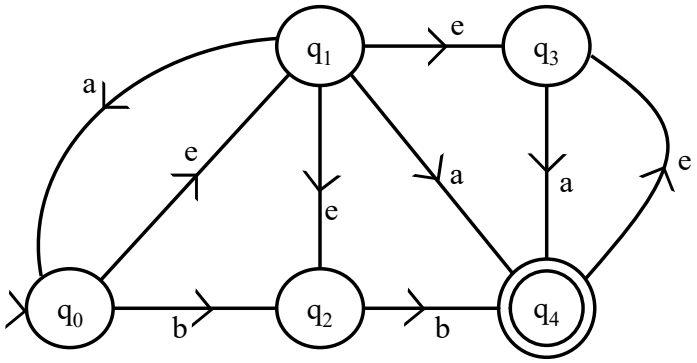
(b) Find the regular expression and construct the finite automaton for the formal language 4

(i) $L = \left\{w \in \left\{a,b\right\}^* : ab \text{ is a substring of } w\right\}$

(ii) $L = \{w \in \{a,b\}^* : a \text{ and } b \text{ occur even number of times in } w\}$

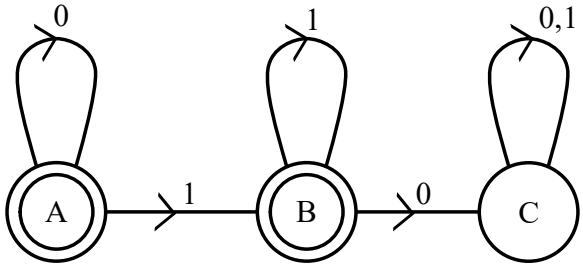
(c) Design a non-deterministic finite automaton (NFA) that accepts strings over $\{a,b\}^*$ which contains a substring *aa* or *bb*. 4

(d) Convert the given non-deterministic finite automata (NFA) to its equivalent deterministic finite automata (DFA). 5



6. (a) Show that intersection of two regular language is also regular. 4

(b) Find the regular expression for the language accepted by the deterministic finite automata (DFA) 5



(c) Show that $L = \{a^p : p \text{ is a prime}\}$ is not regular language. 5

UNIT–IV

7. (a) Define regular context free grammar.
Construct an NFA for the context free grammar (CFG) given by

$V = \{S, A, B, a, b\}; \sum \{a, b\}$

$R = \{S \to bA; S \to aB; A \to abaS; B \to babS; S \to e\}$ 2+3=5

(b) Construct a push down automata (PDA) that accepts the language

$L = \{wcw^R : w \in \{a,b\}^*\}$ 5

(c) Show that $L = \{ww : w \in \{a,b\}^*\}$ is not context free language (CFL). 4

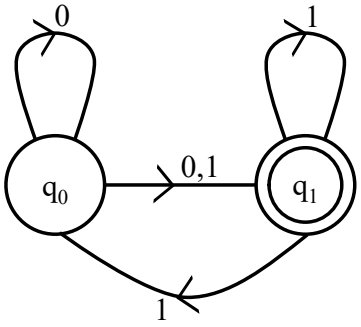
8. (a) Show that a CFL is not closed under intersection. Also show that intersection of CFL and a regular language is a CFL. 2+3=5

(b) Show that CFG $G = (V, \Sigma, R, S)$ where

$V = \{S, a, b, +, *\}, \Sigma = \{a, b, +, *\}, S = S$, and

$R = \{S \to S + S; S \to S * S; S \to a; S \to b\}$ is ambiguous grammar. 4

(c) Determine an equivalent PDA for the NFA given by the diagram 5



UNIT–V

9. (a) Construct a Turing machine which compute the successor function. 4

(b) Define the Universal Turing Machine. 1

(c) Let $L = \{w : aa$ is not a substring$\}$. Construct a Turing machine which accept the given language. 4

(d) Design Turing machine which accepts $L = \{a^n b^n c^n : n \geq 0\}$. 5

10. (a) Define a machine schema. Also draw the copying machine and the right shifting standard machine. 1+4=5

(b) Differentiate between recursive language and recursively enumerable language. Prove that compliment of recursive language is recursive. 2+4=6

(c) Find a post correspondence solution for the given list
M = (110,0011,0110) and N = (110110,00,110). 3
