

**2021**  
**B.A./B.Sc.**  
**Fifth Semester**  
**CORE – 11**  
**STATISTICS**  
*Course Code: STC 5.11*  
 (Stochastic Processes & Queuing Theory)

Total Mark: 70

Pass Mark: 28

Time: 3 hours

Answer five questions, taking one from each unit.

**UNIT-I**

1. (a) Define a probability generating function. Can  $P(s) = \frac{2}{1+s}$  be the probability generating function of a random variable  $X$ ? Give reason. 2+3=5
- (b) Suppose an unbiased dice is thrown 10 times and getting a number less than 3 in each throw is a success. Find the probability generating function of the number of successes and hence find the mean and standard deviation of the number of successes. 3+1+2=6
- (c) Distinguish between discrete and continuous state space. 3
2. (a) If  $P(s)$  is the probability generating function (p.g.f.) of a random variable  $X$ , then find the p.g.f. of (i)  $X+1$  (ii)  $2X$  1+1=2
- (b) If  $X$  and  $Y$  are independently distributed binomial variates with parameters 5, 0.5 and 7, 0.5 respectively, then obtain the probability generating function of  $Z=X+Y$  and hence obtain  $E(Z)$ . 4+2=6
- (c) Let  $X$  be a random variable with probability mass function  $P(X=n) = q^{n-1}p$  for  $n = 1, 2, 3, \dots$ . Obtain the probability generating function of  $X$  and also  $E(X)$  and  $S.D.(X)$ . 6

**UNIT-II**

3. (a) Define the following:  
 (i) Markov Chain

- (ii) Transition probability
  - (iii) Transition probability matrix 2+2+2=6
- (b) Let  $\{X_n, n \geq 0\}$  be a Markov chain with three states 0, 1, 2 and with

transition probability matrix  $\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ . The initial

probability distribution is  $P(X = i) = \frac{1}{3}$ ; for  $i = 1, 2, 3$ . Find

- (i)  $P(X_2=2, X_1=1/X_0=2)$
  - (ii)  $P(X_2=2, X_1=1, X_0=2)$
  - (iii)  $P(X_2=1/X_0=1)$  2+2+2=6
- (c) Distinguish between transient and persistent state. 2
4. (a) Explain absorbing state and irreducible state with the help of example. 2+2=4
- (b) The transition probability matrix of a Markov Chain  $\{X_n, n = 1, 2, 3, \dots\}$  having three states 1, 2, 3 is

$\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$  and the initial distribution is  $\pi_0 = (0.7, 0.2, 0.1)$ .

Find

- (i)  $P(X_2=3)$
  - (ii)  $P(X_2=2, X_0=1)$  6+2=8
- (c) Define an ergodic state. 2

### UNIT-III

5. (a) What is a counting process? Write down four properties of counting process. 2+2=4
- (b) Define and derive Poisson process. 2+8=10
6. (a) Define independent increments and stationary increments of counting

process. When does a counting process become Poisson process?

$$2+2+2=6$$

- (b) Deduce two important properties of Poisson process. 8

#### UNIT-IV

7. (a) What do you understand by a queue? Describe the characteristics of an input process in a queuing system. 2+6=8

- (b) Write short notes on:

(i) Steady state

(ii) Transient state

(iii) Explosive state 6

8. (a) Write a note on service mechanism. 6

- (b) What do you understand by the queuing model  $[M/M/1/(\infty/FIFO)]$ ? A mobile phone repairman thinks that the time spent on his jobs have an exponential distribution with mean of 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is repairman's expected idle time each day? How many jobs are ahead of the average set just brought in? 2+6=8

#### UNIT-V

9. (a) Define demand, supply, demand curve and supply curve. What is equilibrium price? The supply and demand curves of a commodity are given by the equations  $s = 128 + 8p$  and  $d = 478 - 6p$  respectively. Find the equilibrium price. Also find the quantity in equilibrium price exchanged. 4+1+2=7

- (b) What is Engel's law and Engel's curve? The price elasticity of demand curve  $x = f(p)$  is of the form  $(a - bp)$  where  $a$  and  $b$  are given constants. Find the demand curve. 4+3=7

10. (a) Define price elasticity of demand. Find the demand function with constant price elasticity. 3+4=7

- (b) What is the Pareto's law of income distribution? Write a note on curves of concentration. 3+4=7