2021

B.A./B.Sc.

Fifth Semester

CORE – 11

MATHEMATICS

Course Code: MAC 5.11

(Multivariate Calculus)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define continuity of function of two variables $f(x, y)$ at a point (x_0, y_0). Find the value of $f(0, 0)$ so that the function

$$f(x,y) = \ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2 + y^2}\right)$$ is continuous at (0,0). 1+4

(b) Let $f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2}, & (x,y) \neq 0 \\ 0, & (x,y) = 0 \end{cases}$. Show that

$$f_y(x,0) = x \quad \forall x \text{ and } f_x(0,y) = -y \quad \forall y$$ 5

(c) Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ for $w = 2xy$ where $x = s^2 + t^2$ and $y = s/t$ at $(s, t) = (-1, 1)$. 4

2. (a) Define limit of a function of two variables. By using definition of limit show that $\lim\limits_{(x,y)\to(0,0)} \frac{4xy^2}{x^2 + y^2} = 0$. 1+4

(b) By using definition of partial derivatives, evaluate $f_x(-2,1)$ and

$f_y(-2,1)$ for the function $f(x,y)=4+2x-3y-xy^2$ 5

(c) Show that the function $w=\sin(x+ct)$ satisfies one-dimensional wave equation. 4

UNIT–II

3. (a) Find the directions in which the function $f(x,y)=x^2y+e^{xy}\sin y$ increases or decreases most rapidly at $P(1,0)$. Also find directional derivative in these directions. 3+2=5

(b) Find the points on the curve $x^2y=2$ nearest to the origin. 5

(c) Find $div\,\vec{F}$ and $curl\,\vec{F}$ for $\vec{F}(x,y,z)=xz^3\hat{i}+2y^4x^2\hat{j}+5z^2y\hat{k}$ 2+2=4

4. (a) Find absolute maxima and minima for $f(x,y)=2x^2-4x+y^2-4y+1$ on the closed triangular plate bounded by $x=0, y=0$ and $y=2x$ in the first quadrant. 5

(b) Use the method of Lagrange multiplier to find the dimension of a rectangle with perimeter P and maximum area A. 5

(c) Find the equation of the tangent line and gradient of $x^2+y^2=4$ at $\left(\sqrt{2},\sqrt{2}\right)$ 2+2=4

UNIT–III

5. (a) Evaluate $\int_0^2\int_0^1 \frac{x}{1+xy}\,dydx$ 4

(b) Evaluate $\int_0^2\int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dydx$ 5

(c) Evaluate $\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} dz\; rdr\; d\theta$ 5

6. (a) Sketch the region of integration of $\int_0^1 \int_{1-x}^{1-x^2} dydx$ and write an equivalent double integral with order of integration reversed. 4

(b) Evaluate $\int_0^2 \int_0^x ydydx$ by changing into polar form. 5

(c) Evaluate $\int_0^\pi \int_0^\pi \int_r^{2\sin\phi} \rho^2 \sin\phi \, d\rho d\phi \, d\theta$ 5

UNIT–IV

7. (a) Evaluate $\int_C (xy + y + z) ds$ along the curve

$\vec{r}(t) = 2t\hat{i} + t\hat{j} + 2(1-t)\hat{k}, \ 0 \le t \le 1$ 4

(b) Evaluate $\iint_R \frac{y-4x}{y+4x} dA$ by making appropriate change of variables where R is the region enclosed by the lines $y = 4x$, $y = 4x + 2$, $y = 2 - 4x$ and $y = 5 - 4x$. 5

(c) Let $\vec{F} = -4xy\hat{i} + 8y\hat{j} + 2\hat{k}$ be the velocity field of a fluid flowing through a region in space. Find the flow along the curve $\vec{r}(t) = t\hat{i} + t^2\hat{j} + \hat{k}, \ \ 0 \le t \le 2$ in the direction of increasing t. 5

8. (a) Integrate $f(x, y) = (x + y^2) / \left(\sqrt{1 + x^2}\right)$ over the curve $C: y = x^2/2$ from (1, ½) to (0, 0). 4

(b) Define potential function. Find potential function f for the field $\vec{F} = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$ 5

(c) Show that $2xydx + (x^2 - z^2)dy - 2yzdz$ is exact and evaluate

$\int_{(0,0,0)}^{(1,2,3)} 2xydx + (x^2 - z^2)dy - 2yzdz$ 5

UNIT–V

9. (a) Find the area of the portion cut from the paraboloid $x^2 + y^2 - z = 0$ by the planes $z = 2$ and $z = 6$. 4

(b) Verify Green's theorem for the field $\vec{F} = -x^2 y\hat{i} + xy^2 \hat{j}$ by taking the domain of integration to be the disk $R : x^2 + y^2 \le a^2$ and its bounding circle $C : \vec{r}(t) = a\cos t\hat{i} + a\sin t\hat{j}, \quad 0 \le t \le 2\pi$ 5

(c) Use Stokes theorem to evaluate $\oint \vec{F} \cdot d\vec{r}$ if $F = y\hat{i} + xz\hat{j} + x^2\hat{k}$ and C is the boundary of the triangular portion of the plane $x + y + z = 1$ in the first octant, counter clockwise when viewed from above. 5

10. (a) Use Green's area formula to evaluate area of the region enclosed by the circle $\vec{r}(t) = a\cos t\hat{i} + a\sin t\hat{j}, \quad 0 \le t \le 2\pi$ 4

(b) Find the surface area of the sphere of radius *a* by parameterizing. 5

(c) Verify Stokes theorem for the vector field $\vec{F} = 2xy\hat{i} + x\hat{j} + (y+z)\hat{k}$ and the surface $z = 4 - x^2 - y^2,\ z \ge 0$ oriented with unit normal pointing upward. 5