2021
B.A./B.Sc.
Third Semester
CORE – 6
MATHEMATICS
Course Code: MAC 3.21
(Group Theory - I)

PART–B
Total Mark: 30

Answer the following questions.

1. (a) Show that the set $\{1, \omega, \omega^2\}$, where ω is the cube root of unity, forms a group with respect to multiplication. 4
   (b) Prove that the inverse of each element of a group is unique. 2

2. (a) Prove that the centre of a group G is a subgroup of G. 4
   (b) Prove that a cyclic group is abelian. 2

3. State and prove Lagrange's theorem for finite groups. 6

4. (a) Show by an example that we can find three groups $E \subseteq F \subseteq G$, where E is normal in F, F is normal in G but E is not normal in G. 2
   (b) Prove that every quotient group of a cyclic group is cyclic. 4

5. State and prove the fundamental theorem of group homomorphism. 6
