2021
B.A./B.Sc.
Third Semester
CORE – 5
STATISTICS
Course Code: STC 3.11
(Sampling Distributions)

PART–B
Total Mark: 30

Answer the following questions.

1. Suppose an unbiased coin is tossed three times and X denotes the numbers of getting heads. Prove that Chebychev's inequality gives $P\left(\left|X - \frac{3}{2}\right| \geq 3\right) \leq \frac{1}{12}$, while actual probability is zero.　　6

2. (a) Let a random sample of size 5 be drawn from an exponential population $f(x) = \theta\, e^{-\theta x}$; $x \geq 0$, $\theta > 0$. Obtain the probability density function of the sample median.　　3
   (b) Let a random sample of size 5 be drawn from a uniform population with parameters a and b ($a<b$). Obtain the probability density function of the last order statistics.　　3

3. A company has the head office at Kolkata and a branch office at Mumbai. The personnel director wanted to know if the workers at the two places would like the introduction of a new plan of work and a survey was conducted for this purpose. Out of a sample of 500 workers at Kolkata, 62% favoured the new plan. At Mumbai out of a sample of 400 workers, 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level?　　6

4. Define chi-square variate. For a chi-square distribution with n degrees of freedom, establish the following recurrence relation between the moments:
   $$\mu_{r+1} = 2r\left(\mu_r + n\mu_{r-1}\right),\ r \geq 1$$

5. (a) Derive the probability density function of student's t-distribution with n d.f.　　4
   (b) Show that the mode of the F-distribution with $v_1(\geq 2)$ & v_2 d.f. is given by $\dfrac{v_2(v_1 - 2)}{v_1(v_2 + 2)}$ and is always less than unity.　　2