2021
B.A./B.Sc.
First Semester
GENERIC ELECTIVE – 1
MATHEMATICS
Course Code: MAG 1.11
(Calculus)

PART–B
Total Mark: 30

Answer the following questions. 6×5=30

1. (a) Find $\frac{d^2y}{dx^2}$ for the function $x = t - t^4, y = t^2 + t^3$ 3

   (b) If $y = \sin^{-1} x$, prove that 3

   (i) $\left(1-x^2\right)y_2 - xy_1 = 0$

   (ii) $\left(1-x^2\right)y_{n+2} - \left(2n+1\right)xy_{n+1} - n^2 y_n = 0$

2. (a) Discuss the applicability of the mean value theorem
   $f\left(b\right) - f\left(a\right) = \left(b-a\right)f'\left(\varepsilon_1\right), a < \varepsilon_1 < b$ for $f\left(x\right) = x\left(x-1\right)\left(x-3\right)$
   Find ε_1 if the theorem is applicable in $\left[0,4\right]$. 3

   (b) Verify Rolle's theorem in $\left[1,3\right]$ for the function $f\left(x\right) = \log\left(\frac{x^2+3}{4x}\right)$. 3

3. (a) Show that $\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\frac{x^7}{7} + \ldots$ 3

   (b) Evaluate $\lim_{x\to 0}\frac{x^2 e^x}{\cos x - 1}$ using Taylor series. 3

4. (a) Evaluate $\int \frac{dx}{\sqrt{x} - \sqrt{x-1}}$ 2

   (b) Show that $\int_0^{\frac{\pi}{2}} \sin^{\frac{3}{2}} x \cos^3 x\, dx = \frac{8}{45}$ 4

5. Find the area of the first quadrant included between the parabola $y^2 = bx$ and the circle $x^2 + y^2 = 2bx$. 6