2021

B.A./B.Sc.

First Semester

CORE – 2

MATHEMATICS

Course Code: MAC 1.21

(Algebra)

PART–B

Total Mark: 30

Answer the following questions.

1. (a) Find the cube roots of $z = 2 + 2i$ 3

   (b) Find the polar representation of $z = \cos\alpha - i\sin\alpha,\ \alpha \in [0, 2\pi]$ 3

2. (a) Let $A = \{x \in \mathbb{R} \mid x \neq 2\}$ and $B = \{x \in \mathbb{R} \mid x \neq 1\}$. Define $f : A \to B$ and $g : B \to A$ by

   $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{2x}{x-1}$

   (i) Find $(f \circ g)(x)$

   (ii) Are f and g inverse of each other? Explain. 3

   (b) Prove by mathematical induction that $1^3 + 2^3 + ... + n^3 = \dfrac{n^2(n+1)^2}{4}$ for any natural number n. 3

3. (a) Let $u = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ and $w = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$. Show that $3u - 5v - w = 0$. Also find x_1 and x_2 which

   satisfies the equation $\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$ 3

   (b) Write the solution set of the given homogeneous system in parametric vector form:

   $$x_1 + 3x_2 - 5x_3 = 0$$
   $$x_1 + 4x_2 - 8x_3 = 0$$
   $$-3x_1 - 7x_2 + 9x_3 = 0$$
   3

4. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

   $T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$. Find x such that $T(x) = (-1, 4, 9)$ 3

   (b) T is a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$. Show that T is invertible and find a formula for T^{-1} where $T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$ 3

5. (a) If a vector space W has a basis $D = \{d_1, d_2, \ldots, d_n\}$, then show that any set in W containing more than n vectors must be linearly dependent. 3

(b) Find the characteristics polynomial and the eigenvalue of $A = \begin{bmatrix} 5 & -3 \\ -4 & 3 \end{bmatrix}$. 3

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