2021
B.A./B.Sc.
First Semester
CORE – 1
MATHEMATICS
Course Code: MAC 1.11
(Calculus)

PART–B
Total Mark: 30

Answer the following questions. 6×5=30

1. (a) Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$. 3

   (b) Show that if $x \neq 0$ then $y = \dfrac{1}{x}$ satisfies the equation $x^3 y'' + x^2 y' - xy = 0$. 3

2. (a) Draw the graph of $\dfrac{8}{x^2 - 4}$ and label the coordinates of the stationary and inflection points. Also, show horizontal and vertical asymptotes, if any. 3

   (b) A farm determines that x units of its product can be sold daily at p rupees per unit, where $x = 1000 - p$. The cost of producing x units per day is $C(x) = 3000 + 20x$
   (i) Find the revenue function $R(x)$
   (ii) Find the profit function $P(x)$ 3

3. (a) Evaluate $\int \sin^2 x \cos^4 x\, dx$. 3

   (b) Find the volume of the solid that results when the region enclosed by the curves $x = y^2, x = y + 2$ is revolved about the x-axis. 3

4. (a) Rotate the coordinate axes to remove the xy term from $9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$. Then identify the type of conic and sketch its graph. 4

   (b) Find the eccentricity and distance from the pole to directrix for $r = \dfrac{3}{2 - 2\cos\theta}$ and sketch the graph in polar coordinates. 2

5. (a) The position function of a particle is given by $\vec{r} = e^t \hat{i} + e^{-2t} \hat{j} + t\hat{k}; t = 0$. Find
   (i) the scalar tangential and normal components of acceleration at the stated time.
   (ii) the vector tangential and normal components of acceleration at the stated time. 3

   (b) A shell is to be fired from ground level at an elevation angle of 30 degree. What should the muzzle speed be in order for the maximum height of the shell to be 2500 feet? 3